Actuarial Pension Forecasting

Industrial and Applied Mathematics Colloquium Series
Kent State University

Presented by Bill Roberts

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Bill Roberts addressed the colloquium with an application of actuarial mathematics to pension forecasting. Simply put, this is just the problem of determining how much money to invest so that a specified amount is accrued over a given time period. But while the statement of the problem is simple, the implementation of the solution can be much more complicated. What Bill Roberts described in the colloquium is a simplified and idealized situation whose solution illustrates the highlights of the problem.

The pension obligation of a benefit plan is a stream of payments to the beneficiary of the plan. What must be determined is the actuarial present value of all benefits attributed by the plan's benefit formula to meet this obligation. Furthermore, the plan sponsor must determine how to invest the funds required to fulfill the obligation. For example, the sponsor can invest the funds in Treasury bonds with zero risk, or, with the hopes of higher returns and hence a surplus after the obligation is paid, invest in riskier equities. We will assume the former in the following.

A bond is a loan made by an investor to an organization (either a corporation or a governmental body, called the issuer) and is characterized by several features. The par value of a bond is just the numerical face value of the bond. Note that this is not necessarily the purchase price of the bond. Associated with a bond is a coupon, which is an amount paid to the investor at regular intervals (say, once every 6 or 12 months) over the life of the bond to reflect interest on the loan. A maturity date is also associated with the bond. This is the date at which the par value of the bond will be paid by the issuer to the investor. The yield of a bond is the quotient of the coupon amount and the price paid for the bond. An investor may wish to look for a discounted bond (one selling below its par value) to increase the yield of the bond. A bond also has a bond rating. This is an assessment of the risk of investing in the issuer of the bond. (A company, for example, could go bankrupt and default on its debts.) There is less risk in investing in a company with a higher bond rating.

The problem is to decide what bonds to include in a portfolio to meet the pension obligation. Before we do this, however, we need to specify the details of the problem. First, the cash flows from the bonds (which come from coupon and maturity payments) must equal the benefits paid in a given period. (For instance, if benefit payments are made once a month, then the cash flow from the bonds should match these every month. One can easily imagine why an employer would not want to draw assets, even temporarily, from another source.) The cost of the obligation is now the cost of the portfolio, which in turn is the number of bonds of each issue purchased multiplied by its price.

Second, discount rates should be found. This will reduce the amount the pension sponsor needs to invest to meet the obligation. Many so-called discount curves are available for large investments, but a favored one is the Citigroup pension discount curve.

To solve the problem of which bonds to invest in to meet the obligation, one must construct an appropriate portfolio (where the cash flows are equal) that has a minimum cost. To this end, let C be the matrix of cash flows from each bond in each period; let P be the vector of bond prices; let B be the vector of benefit payments; let N be the vector of the number of bonds purchased of each issue. The price of the portfolio is now NP, and the problem is solved by minimizing NP subject to the constraints NC=B and $N \geq 0$. (The last constraint avoids the complication of selling bonds.)

The situation described above is readily solved by using the simplex algorithm, but actual situations need not be linear. For instance, excess cash flows often arise when benefit payments in future years are greater than benefit payments in the current year. In this situation, interest must be credited on the excess cash flow. One method to deal with this is to tread the yield curve (which predicts future interest rates) as reflective of one-year rates at each point in the future. This method preserves the linearity of the system but is deemed unacceptable to auditors because one-year forward rates can be significantly higher than current one-year rates. The auditors suggested that it be assumed that the yield curve is fixed and that interest on excess cash flow be calculated using current and all forward-year rates. This generates more money on paper but destroys the linearity of the system. Furthermore, the yield curve is not fixed. In fact, it changes on a daily basis. So far, no analysis has been done to see how changes in the yield curve affect the predictions.

A benefit sponsor may invest in alternatives. For instance, the bonds mentioned above are a of fixed-yield type. That is, the yield does not change. There is another type of bond, called a floating-yield bond, whose yield fluctuates with the daily market. A sponsor may also opt to invest in equities (stocks). Although equities are much riskier than bonds, they are also potentially much more profitable. A problem facing actuaries is convincing sponsors that there is no difference in current value between the options, but that investing in bonds is the better deal (because tax breaks). The sponsors tend to only see the potential for more money instead of a guaranteed smaller amount. This, however, is a problem of an ignorant perspective.