Example of application of the Robust Mathematical Modeling concepts

Dispersion of satellite debris

The ambition of the RMM program is to handle uncertainties, which occur quite naturally in any scientific project.

Let's describe a specific example : the fall of a satellite debris (this comes from a contract we had we the French "Centre National d'Etudes Spatiales", in 2004 - 2005).

The question is : if a satellite enters into the atmosphere, it will disintegrate ; can one predict the places where the debris will fall ?

The interest of this question comes from the fact that, if the debris are recognized to fall into a populated area, then the satellite should be guided to some other area. But this requires to keep some propellant in it, so as to control the orbit : this is costly.

Originally, the CNES used some software, designed to compute the trajectography of any object in any atmosphere (including, for instance, a vessel sent to Planet Mars). This software must be fed with all information concerning the debris (size, composition, and so on), and all information about atmosphere (density at various altitudes, and so on). Then the computation is quite lengthy (it takes hours).

We explained that this approach was not satisfactory : the use of a precise software in this case is irrelevant, because the data needed in it are largely unknown. One does not know exactly the size and composition of the debris, nor the density of the atmosphere.

Our approach was completely probabilistic. We built a very simple software (in VBA using Excel), taking into account the weight and air resistance. For air resistance, the classical formula is :

$$R = \frac{1}{2} C_x S \rho V^2$$

where C_x is a resistance coefficient, S is the surface of the debris, ρ is the density of the atmosphere and V is the speed of the debris (relative to the air).

In this formula, everything was treated as probabilities, since nothing was really certain. For instance, the air density was taken as a uniform law between $\pm 10\%$ the usual air density (depending on the altitude, we took several values). Even the exponent "2" in V^2 was treated as a random variable (uniform law between 2 and 2.5) because nobody is really sure that the above formula is correct for high speeds (Mach 7) and low pressure (100 km altitude).

The result was a "probability map" : the ground was divided into squares of 100 m x 100 m, and we indicated the probability that a given debris would fall in each square. This was obtain by running several thousands of configurations, with various values for all random variables. We could achieve this very easily, because the software run in fractions of a second.

In fact, we treated the whole disintegration problem : the whole satellite disintegrates ; some of the debris reach the ground, some burn during the fall. We took the whole process into account (even the disintegration was treated in a probabilistic manner, because nobody is really sure of the temperature and of the type of debris). The result was a complete probabilistic map : for a given satellite, we computed the probability that some debris would hit a specific square of the map.

We could even evaluate a cost, which is analogous to an insurance. We assumed that the satellite would disintegrate above Chambéry (a city, East of France) ; we know the density of population. We assumed the following rules : if a person is hit, it will cost one million Euros. We added a nuclear plant, and we assumed that if the nuclear plant was hit, it would cost ten million Euros. The question was : what is the sum required for insurance. The answer was : around 100 Euros.

So, in this case, the benefit of the RMM approach is quite clear : since the data are imprecise, the result needs to be of probabilistic nature. There is no need for a lengthy and precise software, which, after hours of work, brings solutions that are completely artificial, since nobody knows in reality what the data are.