Free enterprise in Mathematics

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In any University, or laboratory, a research program is a result of a long elaboration, requiring consents of participating colleagues, approval by committees, refereing by experts, funding by Agencies. A Ph.D. program, for instance, cannot come into existence just because half a dozen individuals have decided it ; not every State University is allowed to create one.

It does not mean that Ph.D. programs are the property of the government (State or Nation), but, in public Universities, they are controlled and permitted by the government. Is this the government wish ? I do not think so. My impression, from the few contacts I had here and there, is on the contrary that the political authorities wish the scientific community had more initiative. So the "umbrella" of committees, assemblies, and so on, has been elaborated, over the years, by the mathematical community itself : individual initiatives are disregarded as being "pure propaganda for oneself". An action (such as : organizing a small conference) which typically requires three people, will start with a meeting by six ; this meeting decides nothing, but talks to the chairman. The chairman will observe that the idea is nice, but organizing it in place A might offend the tenants of place B, so place B should be associated to the organization, and the meeting itself should be held in place C. And we have not started talking about money, yet ...

Now, if I am a shoemaker, I build shoes. I start with one or two associates, a little amount of money, and a small shop. If my shoes are well designed and please people, my business will grow up ; I may hire a few workers and teach them the job. To hire a young guy, I won't need the French permission of the "Direction des Recherches et Etudes Doctorales", or, in the US, that of the National Research Council. At worst, some inspector may come from time to time, to check that the worker is properly treated. The right to create, to hire, to form, is what is called "free enterprise" in our countries. It exists, it is the basis of our constitutions, and may apply as well to mathematics.

Very few mathematicians exert their rights to free enterprise. We do, at I.C.M. On a very modest level, of course : we are not I.B.M., and we have been in existence for five years only. But last year, we made more than $ 140,000, selling polynomials. This enables us, this year, to hire new people, hopefully treating more contracts.

For most mathematicians, "pure" science cannot bring money ; only "applied" can. Five years of work on the edge between both have shown to me that both can, and pure science does not become less interesting when it is brought to applications. Let me take three very striking examples.

– The representation of many-variable polynomials on hypercubes, devised by I.C.M. under a request from the French Army, in order to permit massively parallel computing, comes directly from a paper by Beauzamy-Bombieri-Enflo-Montgomery (Journ. of Number Theory, 90). But in turn, this hypercube rep-

resentation brought a new point of view on polynomials, at the theoretical level, with a lot of unexpected benefits.

– Algorithms in Symbolic Computation, devised jointly by I.C.M. (under a contract with *Digital Eq. Corp.*) and Kent State University (Paul Wang's group) of course borrow heavily from Number Theory. But in turn, they brought new fascinating questions.

For some years, Per Enflo and myself wrote estimates on polynomials of the form

$$\|PQ\| \ \geq \ \lambda \, \|P\|.\|Q\|$$

and people said " It's Number Theory".

But after a while, we wrote instead

$$\|P\| \cdot \|Q\| \ \leq \ \frac{1}{\lambda} \, \|PQ\|$$

and it became "A priori bounds for polynomial decompositions", an essential feature in Symbolic Computation.

– Many questions of Control Theory for linear systems are nothing but Operator Theory. If you study the iterates of a point under a linear operator, $T^n x$, that's Operator Theory. But if, at each stage, you can apply a command, say $(T^n \pm U)x$, that's Control Theory. The bad thing is that the experts of one area do not talk to the others : they even ignore their existence. I wrote a book in 88 about Operator Theory ; Control Theory does not appear anywhere in it.

So my claim is that there is no conceptual difference between pure and applied mathematics. All the results we obtained under our contracts have been published in international journals (except of course a few classified things dealing with specific requests) ; all the contracts, so far, have been renewed, which seems to indicate that the contrators are happy with what they get. We have even signed recently a bigger contract with *Digital*, dealing with Symbolic Computation.

Why does *Digital* (who has some financial problems, like most Computer Companies) care about mathematical research ? Because they think that a good mathematical software will help sell their computers : more people, both in the Industry and in scientific research, will buy these computers if they are properly equipped. But to design mathematical software is a non- trivial task, requiring elaborate mathematical tools, and professional mathematicians. I completely share Digital's point of view on these matters ; I think, moreover, than symbolic computation will be an efficient tool for mathematical research, when it reaches a better degree of development.

Those who give the money want results for this money, and they want results which fit their needs, and within a fixed amount of time. A contract is not a grant. We can't say "we found nothing, but we plan to go on". If instead of what they ask, we come up with a nice theory of the third dual of non-separable quasi-barrelled spaces, we might lose a substantial proportion of our contracts. So people might say that we are not free of what we are doing : we have to adjust to customer's demands, which might not include these nice duals. That's true : we have to produce shoes that fit the demand, not curly, spring-like, high heels.

However, two considerations are important. The first one is that there is room enough for true mathematical research in what we are doing, so we are happy with it. The second one is that the elaborate objects which are usually regarded as important by the mathematical community might very well be artificial questions, disconnected from any type of reality, and allowing a little amount of free enterprise into mathematics might cut down the dominating influence of some academic groups, and bring it closer to some reality.

Everything here is indeed a matter of balance ; fundamental research with no application in mind is of course essential : nobody knows how tomorrow's tools will be built, and quite often abstract discoveries lead to concrete realizations. But it is the researcher's duty to explain that what he is doing is potentially

useful : not this small result here, but the whole area. This social duty is so poorly met by mathematicians that a lot of engineers – not to speak about politicians – think that "there is nothing left for discovery in mathematics".

The usefulness of mathematicians is only perceived through their teaching load, and this also is unfortunate. The connection between teaching and research is only historical, and strongly detrimental to our profession. As a shoemaker, I make shoe, I am not going from one place to the other explaining how to make shoes. Once in a while, I hire a young guy and teach him so he can help me, but that's all.

I have nothing against teaching, nor do I against plumbing or car repair : someone has to do it, not necessarily me. Basically, the government considers mathematicians as teachers because it does not know how to consider them otherwise : the social usefulness of the profession is not perceived. Once I met the Counselor for Education of the French Prime Minister, and he strongly supported my views, saying "mathematicians, so far, have not been clever enough to create their own status". He added, smiling "you might have trouble with the Ministry of Education, because they are narrow-minded".

I would even be glad to teach, as part of a contract signed for instance with some University : we would give a specialized course, on a specific topic ; the same way as, for instance, a stock exchange expert may in a while deliver a course about finances, because he is invited to do so by some institution.

Besides contracts, a great part of our activity during these last two years has been devoted to formation : trying to explain to young people that mathematics are not dead, and can lead to interesting careers, that a Ph.D. in mathematics, no matter what orientation you take later, gives a formation of high quality : being in contact with research is a good preparation in a world which is constantly changing. We organized several conferences in the "Palais de la Découverte" in Paris, and, last June, we held during four days a seminar called "Discovery of Research", in which 30 selected graduate students were able to live with professional mathematicians, ask questions, and hear lectures. Bringing new people to mathematics is a task we regard as essential, and too often neglected by official institutions, despite the fact that the average age is very high among researchers.

I read once an interesting point by Konrad Lorentz, a former Nobel Prize in Medicine, who died recently. He said he had doubts about the long-term survival of some species, such as stags, because the standards which are ranked highest among them are not those which ensure the survival of the species. Indeed, the best considered male is the one with biggest antlers, but these antlers precisely prevent the animal from running fast in case of danger. This, I think, applies very well to mathematicians : among us, we praise the quality of technical achievements ; outside, the quality of teaching is what matters. None of them has anything to do with the survival of the profession. Better be a shoemaker : this might last as long as mankind has legs.

I am not, by any means, trying to give lessons and explain to others what they should do. When you put a sign near your door saying "Vote for Smith for the US Senate", your express an opinion, and you hope it will be followed by others. We, at I.C.M. do not fear competition, but we think we would be in better shape if more people shared our views. As one of my friends said : "Nobody goes into a city where there is just *one* restaurant".